EXERCISE – II HINTS & SOLUTIONS

Sol.1 A,C

Let $A = \begin{bmatrix} a & b & (a\alpha - b) \\ b & c & (b\alpha - 1) \\ 2 & 1 & 0 \end{bmatrix}$

$|A| = 0$

$\begin{vmatrix} a & b & a\alpha - b \\ b & c & b\alpha - c \\ 2 & 1 & 0 \end{vmatrix} = 0$

$C_3 \to C_3 + C_2$

$\begin{vmatrix} a & b & a\alpha \\ b & c & b\alpha \\ 2 & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow a\{c - b\alpha\} - b\{b - 2b\alpha\} + a\alpha\,(b - 2c) = 0$

$\Rightarrow ac - ab\alpha - b^2 + 2b^2a + ab\alpha - 2ac\alpha = 0$

$\Rightarrow (ac - b^2) - 2\alpha\,(ac - b^2) = 0$

$\Rightarrow (1 - 2\alpha)\,(ac - b^2) = 0$

either $1 - 2\alpha = 0$ or $ac - b^2 = 0$

$$a = \frac{1}{2} \text{ or } b^2 = ac$$

Sol.2 A,C

Given A is a square matrix, Let it be equals to

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A' = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Now, $AA' = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & ac + bd \\ ac + bd & c^2 + d^2 \end{bmatrix}$

AA' is symmetric

Sol.3 A,B,C

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ & $\Delta = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ a_{21} & b_{22} & c_{23} \\ a_{31} & b_{32} & c_{33} \end{vmatrix}$

$D.\Delta^T = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}\begin{vmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ a_{13} & b_{23} & c_{33} \end{vmatrix} = \begin{vmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{vmatrix} = D^3$

$\Delta^T = D^2 = \Delta$

If $D = 27$

$\Delta = (3^3)^2 = 36$

$\Rightarrow$ Δ is present cube

Sol.4 A,C,D

Given B is an idempotent matrix & $A = I - B$

Since is idempotent so, $B^2 = B$

Consider $A = I - B$ ....(1)

Post multiply both sides by 'B' in (1)

So, $AB = IB - B^2 = B - B$ $(\because\ B^2 = B)$

$AB = 0$

Premultiply both sides by 'B' in (1) :

$BA = BI - B^2 = B - B$

$BA = 0$

Premultiply both sides by A in (1)

$A^2 = A$ $(\because\ AB = 0)$

Sol.5 A,B

$A' = A^{-1}$

$AA' = I \Rightarrow (AA')^{-1} = I^{-1} = A^{1-1}A' = I$

$A^{1-1}A' = I \Rightarrow A'$ is orthogonal

Now, $A' = A^{-1} \Rightarrow (A')' = (A^{-1})^1$

$A = (A^{-1})^1 \Rightarrow A = (A^1)^{-1}$

$A^{-1}, A = A^{-1}\ (A^1)^{-1}$

$I = A^{-1}(A^{-1})^{-1}$ $(\because A^1 = A^{-1})$

$\Rightarrow$ A^{-1} is orthogonal

Sol.6 B,C

Given $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

As are know that $|A^{-1}| = \frac{1}{|A|}$

For $|A| \neq 0$, So,

$|A^{-1}| = 1(2) + 1\,(0) + 0 = 2$

$\frac{1}{|A|} = 2 \Rightarrow |A| = 2 \Rightarrow$ A is non singular

Now $A^{-1} = |A| \,.\, A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -1 & \frac{1}{2} \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$

Sol.7 B,C

$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$A^2 - 4A + 5I_2 = 0$ ....(1)

LHS $A^2 = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$

$4A = \begin{bmatrix} 4 & -4 \\ 8 & 12 \end{bmatrix}$, $5I_2 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

Put in (1), LHS = RHS

Now,

$A - B = \begin{bmatrix} 1-\alpha & 0 \\ 0 & -2 \end{bmatrix}$ = Diagonal matrix

Sol.8 A,B,C,D

Let $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ Clearly

its a symmetric matrix

Its adjoint matric = $\begin{bmatrix} 4 & -3 \\ -3 & 1 \end{bmatrix}$

which is also a symmetric matrix

So, [A] is correct.

Now let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ which is a unit matrix

Its adjoint matrix = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ which is also a unit matrix

So, [B] is correct.

also, A (adj A) = (adj A) A. Its a property

Now let $A= \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ which is a diagonal matrix

Adjoint of $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ which is a diagonal matrix

So, [D] is also correct

Sol.9 A,C

Given that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $(bc \neq 0)$

satisfies $x^2 + k = 0$

$A^2 + k = 0$

$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & a^2+bc+bd \\ ac+cd & bc+d^2 \end{bmatrix}$

$|A| = ad - bc$

from relation, $\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = 0$

$a^2 + bc + k = 0$ & $\quad$ B (a + d) = 0

$a^2 + bc + k = 0$ $\quad$ C(a + d) = 0

$bc\,(a + d)^2 = 0$

$(a + d)^2 = 0$ $\quad (\because \ bc \neq 0)$

$a + d = 0$

Now from $d^2 + bc + k = 0$

$k = -(d^2 + bc)$

$k = (d^2 + bc)$

$k = -(d \,.\, d + bc) = -(-ad + bc)$

$k = ad - bc = |A|$

Sol.10 A,B

$\phi_1(x) = x + a_1$

$\phi_2(x) = x^2 + b_1x + b_2$

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \phi(x_1) & \phi_1(x_2) & \phi_1(x_3) \\ \phi_2(x_1) & \phi_2(x_2) & \phi_2(x_3) \end{vmatrix}$

$C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x_1+a_1 & x_2-x_1 & x_3-x_1 \\ x_1^2+b_1x_2+b_2 & x_2^2-x_1^2+b(x_2-x_1) & x_3^2-x_1^2+b_1(x_3-x_1) \end{vmatrix}$$

$$= (x_2-x_1)(x_3-x_1)\begin{vmatrix} 1 & 0 & 0 \\ x_1+a_1 & 1 & 1 \\ x_1^2+b_1x_1b_2 & x_2+x_1+b_1 & x_3+x_1+b_1 \end{vmatrix}$$

$$= (x_2-x_1)(x_3-x_1)\begin{vmatrix} 1 & 0 & 0 \\ x_1+a_1 & 1 & 1 \\ x_1^2+b_1x_1+b_2 & x_2+x_1+b_1 & x_3+x_1+b_1 \end{vmatrix}$$

$= (x_2 - x_1)(x_3 - x_1)\{x_3 + x_1 + b_1 - x_2 - x_4 - b_1\}$
$= (x_1 - x_2)(x_2 - x_3)(x_3 - x_1)$

Sol.11 A,B,C,D

Given a_1, a_2, a_3 in A.P.
& b_2, b_2, b_3 in H.P.

$$\Delta = \begin{vmatrix} a_1-b_1 & a_1-b_2 & a_1-b_3 \\ a_2-b_1 & a_2-b_2 & a_2-b_3 \\ a_3-b_1 & a_3-b_2 & a_3-b_3 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} a_1-b_1 & a_1-b_2 & a_1-b_3 \\ a_2-a_1 & a_2-a_1 & a_2-a_1 \\ a_3-a_1 & a_3-a_1 & a_3-a_1 \end{vmatrix}$$

$\Delta = 0$

Sol.12 A,B

Given $\begin{vmatrix} x & 2y-z & -z \\ y & 2x-z & -z \\ y & 2y-z & 2x-2y-z \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} x & 2y-z & -z \\ -(x-y) & 2(x-y) & -z \\ -(x-y) & 0 & 2(x-y) \end{vmatrix}$$

$$= (x-y)^2\begin{vmatrix} x & 2y-z & -z \\ -1 & 2 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$= (x - y)^2\, 4(x + y - z)$
$= 4(x - y)^2 (x + y - z)$

Sol.13 A,B

Given $\Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a+b & (a+b)^2 \\ 0 & 1 & 2a+3b \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$

$$\Delta = \begin{vmatrix} a & 0 & 0 \\ 1 & a+b & (a+b)^2 \\ 0 & 1 & 2a+3b \end{vmatrix}$$

$\Delta = a\,[(a + b)(2a + 3b) - (a + b)^2]$
$\Delta = a(a + b)(2a + 3b - a - b)$
$\Delta = a(a + b)(a + 2b)$

Sol.14 A,B,C

Given a, b > 0 and $\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} a+b-x & a+b-x & a+b-x \\ b & -x & a \\ a & b & -x \end{vmatrix}$$

$$\Delta = (a+b-x)\begin{vmatrix} 1 & 1 & 1 \\ b & -x & a \\ a & b & -x \end{vmatrix}$$

$\Delta = (a + b - x)\,[1(x^2 - ab) - 1(-bx - a^2) + 1(b^2 + ax)]$
$\Delta = (a + b - x)\,[x^2 - ab + bx + a^2 + b^2 + ax]$
$\Delta = (a + b - x)\,[x^2 + (a + b)x + a^2 + b^2 - ab]$
If a = b then $\Delta = (2b - x)(x + b)^2$
If $\Delta = 0$ which gives three real roots.